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USE OF SHEAR LAG FOR COMPOSITE MICROSTRESS

ANALYSIS - LINEAR ARRAY

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The task of finding shear distributions in damaged composites analytically by means of the theory of elasticity is prohibitively difficult, so approximate techniques are generally employed. The most widely used is shear lag theory. This theory contains a parameter representing the shear coupling between fibers that has been difficult to evaluate in the past. The present paper evaluates this parameter for a linear array of fibers. In the limiting case of small fiber diameter to fiber spacing ratio

20.   Abstract

and large matrix thickness to fiber diameter ratio an analytical solution is possible and is provided in this paper. For other geometries the solution is obtained experimentally. It is found that the assumption usually made for the parameter is rather far off for most geometries of practical interest.

NOMENCLATURE

d	distance between fiber or rod centers
d_2	distance between nearest fiber pair in a rectangular array
$\left(\dfrac{h}{d}\right)_{c,v}$	h/d for the interaction between nearest fiber pair in a rectangular array
r_o	radius of a fiber or a rod
t	lamina thickness
w_i	axial displacement of the i'th fiber
z	axial coordinate
F_i'	load transfer to the i'th fiber per unit length
F_o'	axial line force per unit length
G	shear modulus of the matrix
Gh	effective shear stiffness of the composite
I	current generated by the power supply
D_o	a large distance approaching infinity at which the displacement of the field point is essentially zero
R	resistance
α	ratio of the distance between next-to-nearest fiber pair to the

distance between nearest fiber pair in a rectangular array

ρ_e resistivity of the electrolyte

ϕ electrical potential

INTRODUCTION

Studies of the micromechanics of failure of unidirectionally reinforced composites are based on theoretical analyses of the stress distribution in damaged composites under load. Any use of the theory of elasticity for this purpose turns out to be prohibitively difficult, and therefore approximate techniques must be employed. A number of authors have used shear lag theory or some modification thereof for this purpose [1-5]. However the relation between the solutions found in this manner and experimental results has been made uncertain by the fact that the solutions involve a parameter the values of which were not really known. This parameter is the effective shear stiffness of the matrix, which serves to transfer direct stress from one fiber to another.

Several previous papers deal with various aspects of this question. One gives an exact solution for the shear interaction between two infinitely long rigid fibers immersed in an infinite elastic medium [6]. Another gives an approximate solution for the interaction between neighboring pairs of fibers in a square array [7]. A third extends this solution to the case of a rectangular array [8].

The present paper considers a linear array, i.e., a unidirectionally reinforced lamina or tape. A theoretical solution is given for such an array that is valid when the fiber diameter is small compared to fiber separation and fiber separation is small compared to matrix thickness. For other geometries, the solution is obtained experimentally using an electric analogue technique [8]. These findings are compared to assumptions that have been made concerning the shear transfer stiffness by previous investigators.

1

Analytical Solution for Linear Array

The analysis of a linear array can be carried out by considering the case of a three-fiber interaction in which static equilibrium is maintained as in Figure 1. The center fiber is loaded with an axial line force $2F_o'$ per unit length, while the neighbors are each loaded with F_o', directed in the opposite sense.

The shear lag equation becomes

$$F_a' = G\left(\frac{h}{d}\right) (-w_c + 2w_a - w_b) \tag{1a}$$

$$= 2F_o' \tag{1b}$$

Since $w_b = w_c$

$$F_a' = 2G\left(\frac{h}{d}\right)(w_a - w_b) \tag{2}$$

The displacement of each fiber can be obtained by superimposing the effects of all of the applied loads. Employing the relationship between the load points and the field point in [6],

$$w_a = -\frac{F_a'}{2\pi G} \ln (r_o/D_o) - \frac{2F_b'}{2\pi G} \ln(d/D_o)$$

$$= \frac{2F_o'}{2\pi G} \ln(d/r_o) \tag{3}$$

and

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$$w_b = - \frac{F_b{}'}{2\pi G} \ln(r_o/D_o) - \frac{F_a{}'}{2\pi G} \ln(d/D_o) - \frac{F_c{}'}{2\pi G} \ln(2d/D_o)$$

$$= \frac{F_o{}'}{2\pi G} \ln(2r_o/d) \tag{4}$$

Subtracting (4) from (3), we obtain

$$w_a - w_b = \frac{F_o{}'}{2\pi G} \left[2 \ln(d/r_o) + \ln(d/2r_o) \right] \tag{5}$$

Thus,

$$2(w_a - w_b) = \frac{F_a{}'}{2\pi G} \left[3 \ln(d/r_o) - \ln 2 \right] \tag{6}$$

$$\text{or} \quad F_a{}' = \frac{2\pi[2G(w_a - w_b)]}{[3 \ln(d/r_o) - \ln 2]} \tag{7}$$

Comparing (7) with (2),

$$\frac{h}{d} = \frac{2\pi}{3 \ln(d/r_o) - \ln 2} \tag{8}$$

Experimental Setup

The theory underlying the electric analogue can be found in [8]. A linear array with various thicknesses of matrix material was simulated by placing two vertical insulating panels with height exceeding the depth of the fluid (electrolyte), on top of a submerged panel with holes that match the size of the conducting rods and with spacing approximate to the r_o/d under

3

investigation. The insulating panels were spaced at equal distances from the center lines of the rods. The distance was chosen to represent the desired composite thickness. Thus, the fluid between the vertical panels was analogous to the matrix associated with that particular array.

Linear arrays with different thicknesses and rod spacing were investigated, the first experiment involving three rods as in figure 2. A constant AC potential was applied across rods a and b with b connected to c. The current through the power supply was measured in each case. The potential difference between a and b was also measured. The governing equation can be written as

$$\frac{dI}{dz} = \frac{1}{\rho_e} \left(\frac{h}{d}\right) \left(2\phi_{ab}\right)$$

(9)

The effect of the presence of a free rod adjacent to b and c was investigated by a second experiment as in Figure 3.

Results

The results of the experiments for evaluating h/d are summarized in Figures 4-6. In these figures h/d is plotted against r_o/d for various ratios of fiber radius to lamina thickness, r_o/t. We do not have a theoretical solution except for $r_o/t \rightarrow 0$, so Figures 4-6 are based entirely on the experimental data. Extrapolation beyond the data points is aided by the knowledge that $h/d \rightarrow 0$ for $r_o/d \rightarrow 0$, and $h/d \rightarrow \infty$ when $r_o/d \rightarrow 0.5$. These limiting cases are discussed in Reference 7 and 8.

Figure 7 shows the results for h/d for a linear array of rods in an open tank (i.e., the vertical panels are removed, so the matrix has the shape of the tank). Also shown are the theoretical results for an infinite tank. We note that, as expected, the theoretical results agree very closely with the experimental data obtained in a large but finite tank for small values of r_o/d, i.e., where the theoretical solution is valid.

Another limiting case is obtained by noting that for a very rectangular array ($a \gg 1$) the nearest fiber interaction should be almost the same as though the adjacent lamina was not there. Table 1 gives the h/d obtained for $a = 10.5$ and shows that the results are substantially the same as those obtained for a linear array in an open tank.

In this investigation, most of the data were obtained using three rods as in Fig. 1. A linear array is better approximated by a very large number of rods, but a very large number of rods will in many cases exceed the capacity of the tank. To see whether additional rods are really needed, the effect of adding an additional rod at each end as in Fig. 3 was investigated. The added rods were left free to adopt the local potential. The results are contained in Table 2. It was found that the resistance was virtually unaffected by the presence of the additional rods. The largest change was a couple of percent, and it occurred when r_o/d and r_o/t were both large. If either was small, the change in resistance was negligible. We conclude that the results obtained using three rods are sufficiently accurate for most engineering purposes.

Discussion

Some investigators who have employed shear lag to find stress distribu-
tions in single ply composites have assumed that h/d is equal to t/d,
irrespective of the value of r_o/d. Since $t/d = (t/r_o)(r_o/d)$, a plot of t/d
vs. r_o/d is a straight line of slope t/r_o. Such lines are shown in dot-dash
format for $r_o/t = 0.107$, 0.144, and 0.380. We see that the assumption that
$h/d = t/d$ is not bad for most of the r_o/d range when $r_o/t = 0.144$, but is
rather far off for values of r_o/t differing widely from this value.

This paper and its predecessors [7,8] have attempted to find the shear
interaction to be used under the simplifying assumption generally employed in
shear lag theory that only nearest-neighbor fibers interact. The accuracy of
this basic assumption is currently unknown.

6

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         Microstress Analysis - Rectangular Array," submitted to AIAA con-
         currently with the present paper.

Table 1   Comparison of a linear array with a very rectangular
          array with same ratio of fiber radius to fiber spacing.

$\dfrac{r_o}{d}$ $\dfrac{r_o}{d_2}$	0.370	0.325	0.280	0.230
$\dfrac{h}{d}$	3.477	2.724	2.263	1.857
$\left(\dfrac{h}{d}\right)_{c,v}$	3.535	2.712	2.214	1.830

Table 2. Resistance due to 3-fiber interaction and 5-fiber interaction

$\frac{r_o}{d}$ $\frac{r_o}{t}$	0.380	0.310	0.260	0.197	0.144	0.107
0.370	R_3^a=98.40	R_3=97.77	R_3=86.14	R_3=83.17	R_3=81.14	R_3=79.67
	R_5^b=98.90	R_5=97.28	R_5=86.81	R_5=85.30	R_5=81.14	R_5=79.67
0.325	R_3=132.67	R_3=119.64	R_3=109.23	R_3=103.23	R_3=98.12	R_3=95.63
	R_5=134.32	R_5=121.23	R_5=113.07	R_5=102.16	R_5=97.92	R_5=95.73
0.280	R_3=193.08	R_3=169.49	R_3=149.55	R_3=135.54	R_3=127.56	R_3=122.84
	R_5=192.88	R_5=171.28	R_5=149.55	R_5=135.41	R_5=127.56	R_5=122.98
0.230	R_3=248.50	R_3=222.00	R_3=196.08	R_3=169.49	R_3=157.14	R_3=150.00
	R_5=250.25	R_5=222.22	R_5=195.88	R_5=169.49	R_5=157.30	R_5=150.00
0.186	R_3=369.63	R_3=321.61	R_3=269.19	R_3=231.63	R_3=199.40	R_3=187.55
	R_5=369.63	R_5=321.61	R_5=268.92	R_5=231.63	R_5=199.00	R_5=187.74
0.140	R_3=526.32	R_3=475.24	R_3=398.80	R_3=331.67	R_3=261.58	R_3=236.67
	R_5=526.32	R_5=475.24	R_5=399.20	R_5=331.67	R_5=261.58	R_5=236.67
0.090	R_3=993.00	R_3=827.50	R_3=621.25	R_3=496.50	R_3=413.75	R_3=342.07
	R_5=993.00	R_5=827.50	R_5=621.25	R_5=496.50	R_5=413.75	R_5=342.07

a   Resistance of 3-fiber interaction (Ohms)
b   Resistance of 5-fiber interaction (Ohms)
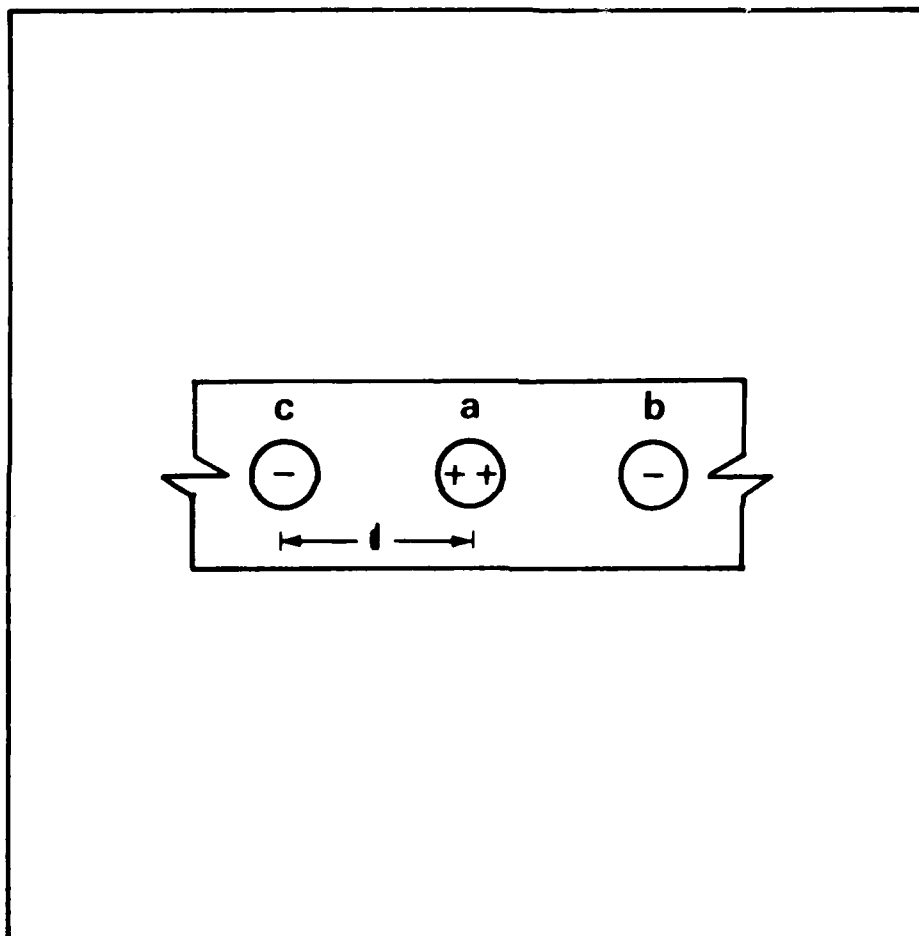
FIG. 1   A 3-FIBER INTERACTION IN WHICH STATIC
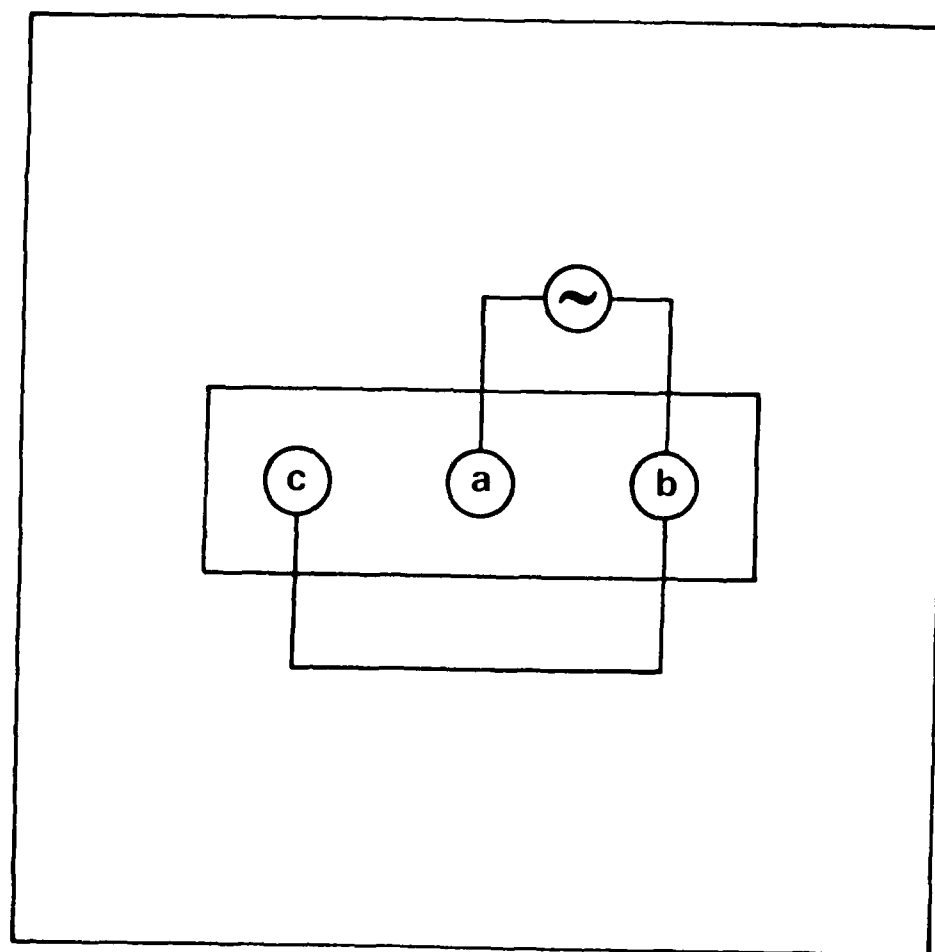   EQUILIBRIUM IS MAINTAINED

FIG. 2   SIMULATION OF A 3-FIBER INTERACTION BY
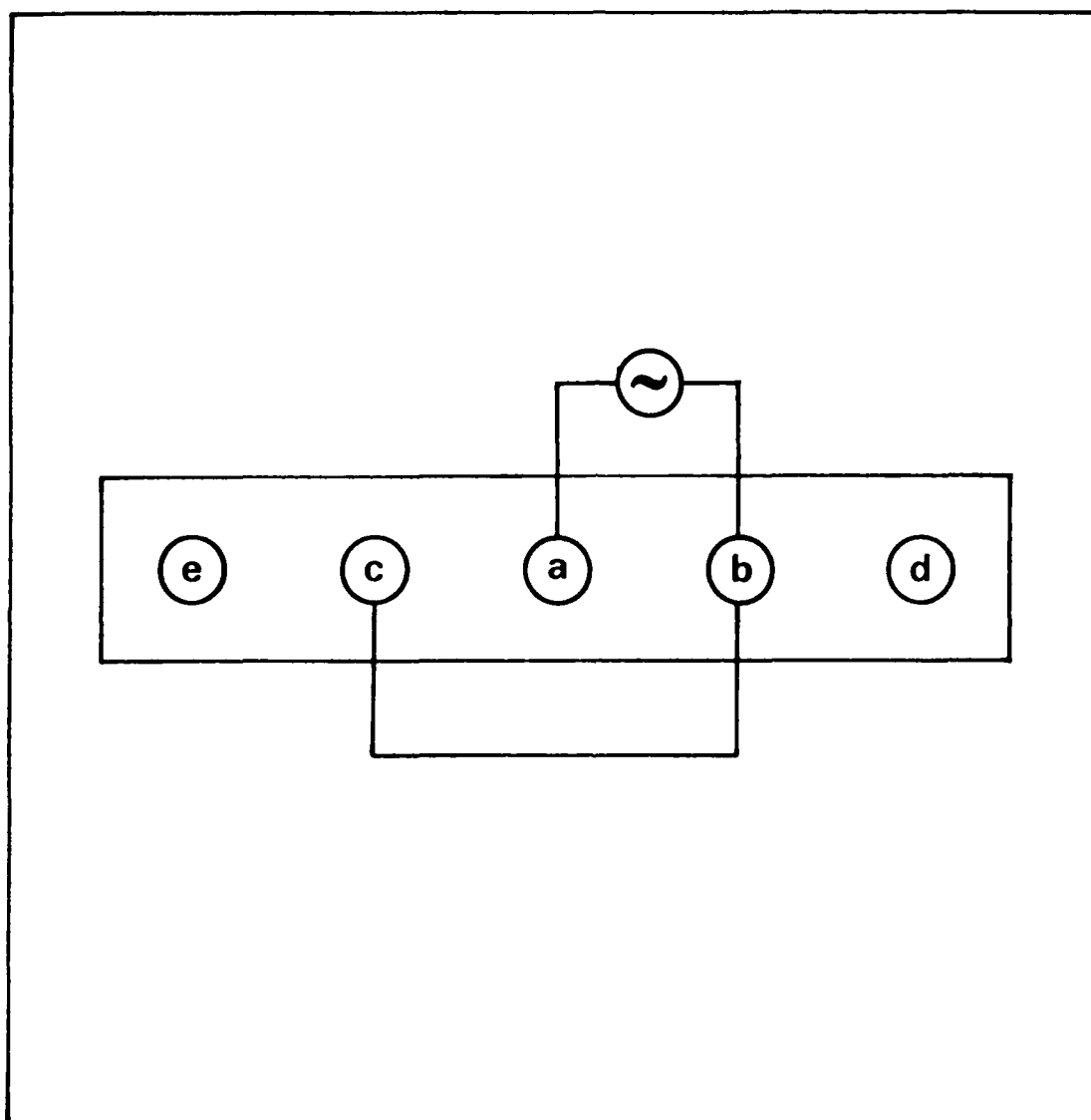AN ELECTRIC ANALOGUE

FIG. 3   SIMULATION OF A 5-FIBER INTERACTION BY AN ELECTRIC
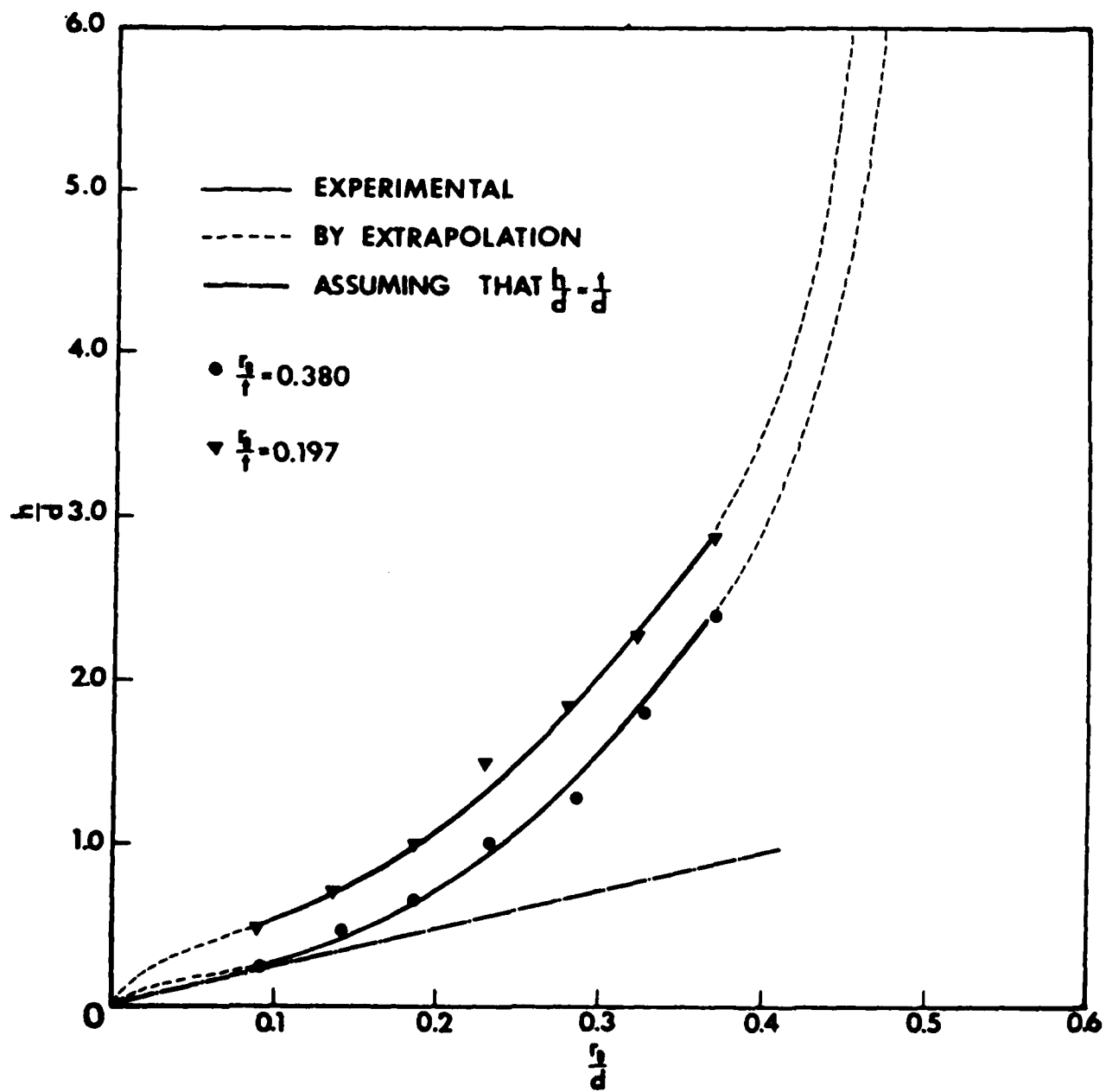ANALOGUE

FIG. 4 $\frac{h}{d}$ FOR LINEAR ARRAY OF FIBERS
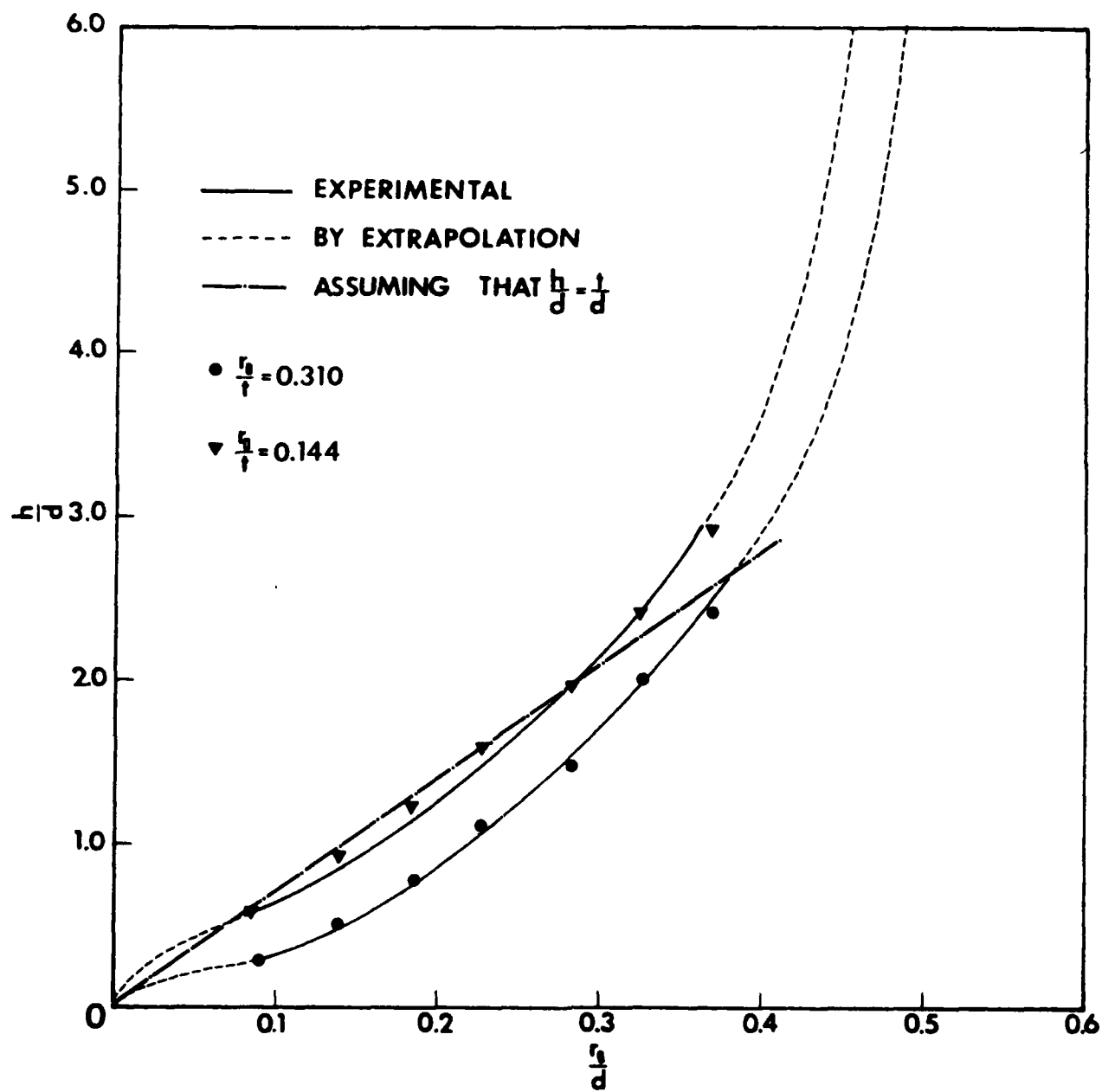
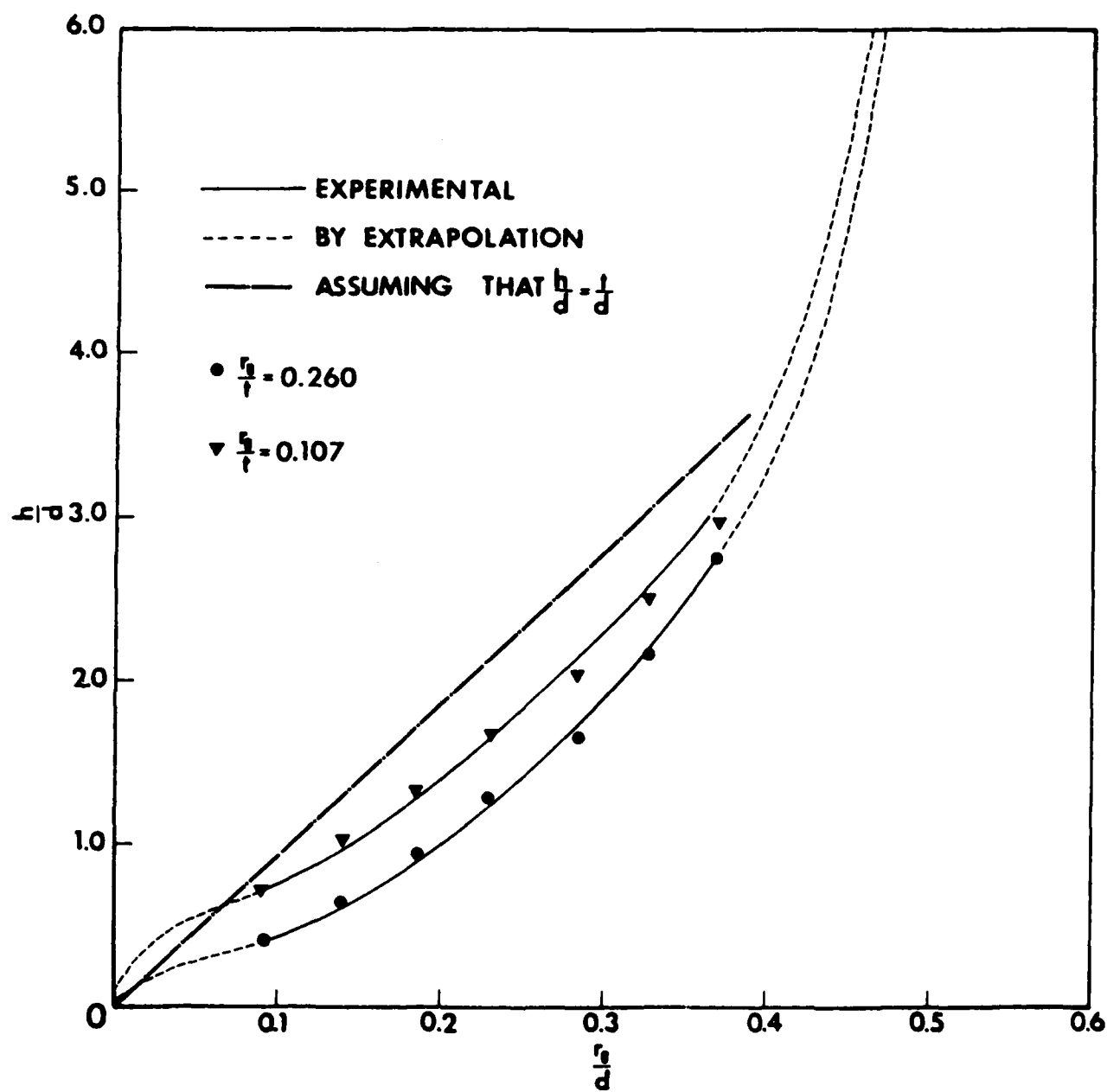FIG. 5 $\frac{h}{d}$ FOR LINEAR ARRAY OF FIBERS
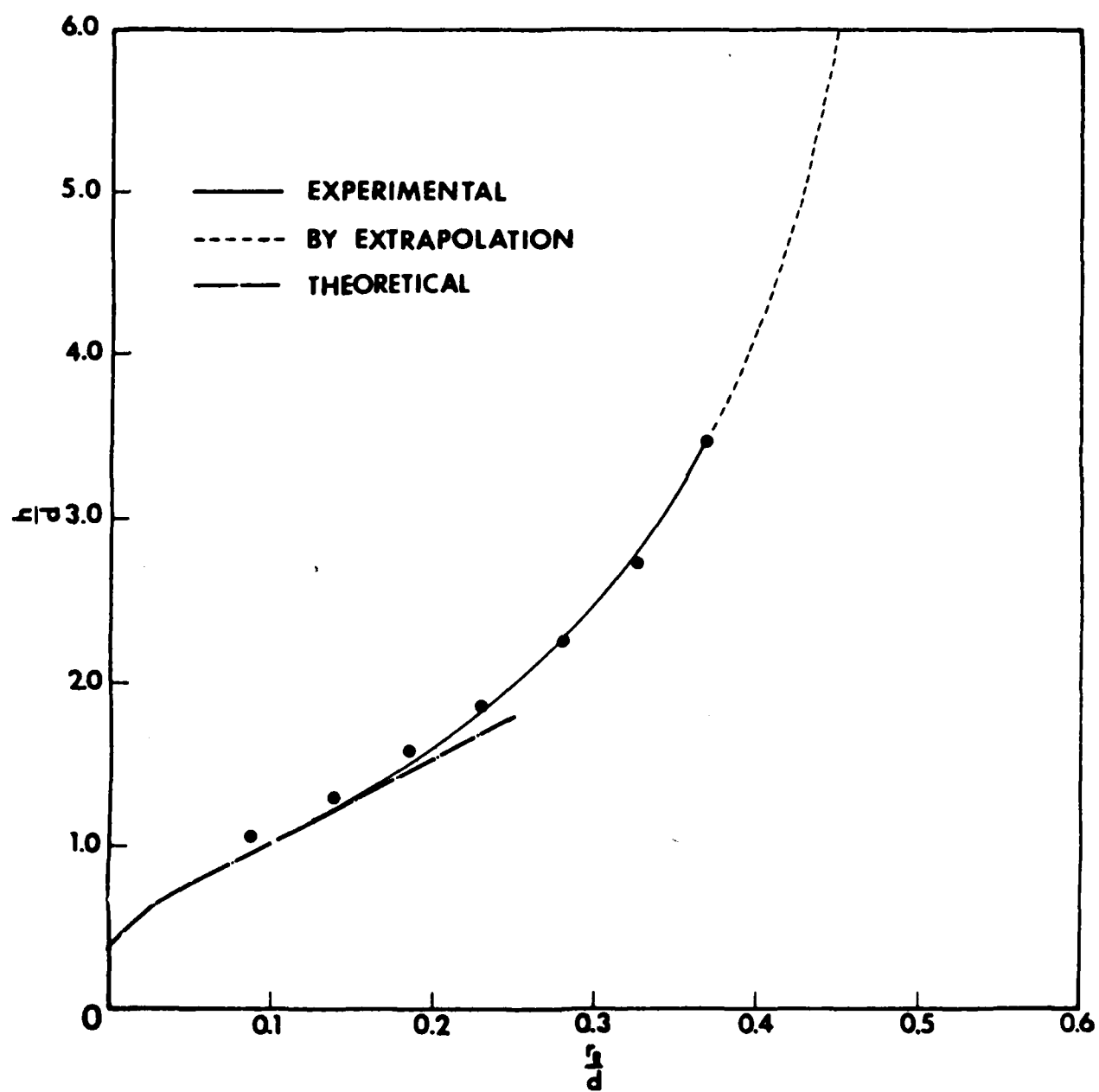
FIG. 6 $\frac{h}{d}$ FOR LINEAR ARRAY OF FIBERS

FIG. 7 $\frac{h}{d}$ FOR LINEAR ARRAY OF FIBERS (OPEN TANK)